
Broadcast Capacity in Wireless Multihop Networks (Mobicom'06)

Alireza Keshavarz-Haddad

Rudolf Riedi

Rice University

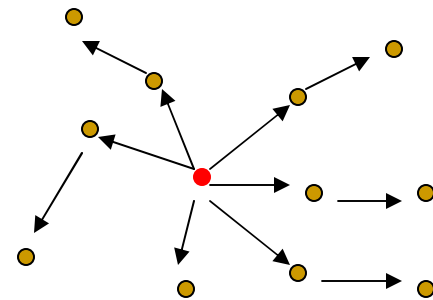
Vinay Ribeiro

I.I.T. Delhi

WISARD'08

Motivation

- Broadcast is an integral part of wireless networks
 - Route discovery
 - Query
 - Alarms and different services



- **Fundamental limits** for broadcast important
 - We compute **capacity** of arbitrary network for delivering broadcast packets
-

Outline:

1. Models and basic concepts
 2. Broadcast capacity
 3. Maximum throughput of broadcast schemes
 4. Unicast capacity vs. Broadcast capacity
-

1. Models and Basic Concepts

Wireless Channel Model

■ Protocol Model:

- **Transmission range:** R
- **Interference range:** $(1+\Delta)R$
- A transmission from X_i to X_j is **successful** iff

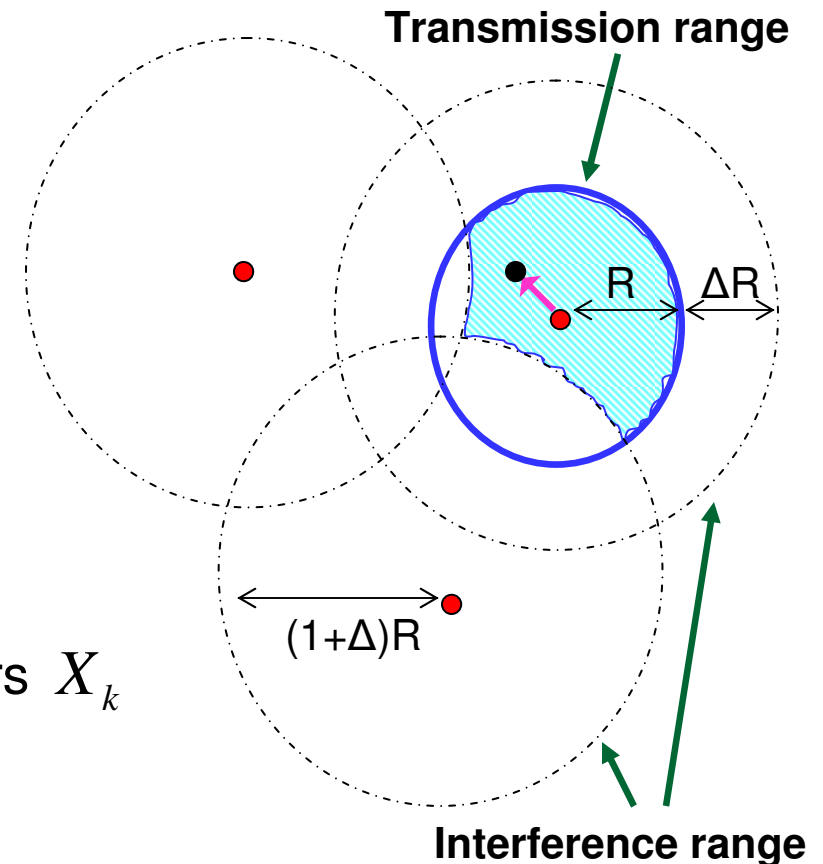
- $|X_i - X_j| \leq R$

- $|X_k - X_j| \geq (1+\Delta) \cdot R$

for all simultaneous transmitters X_k

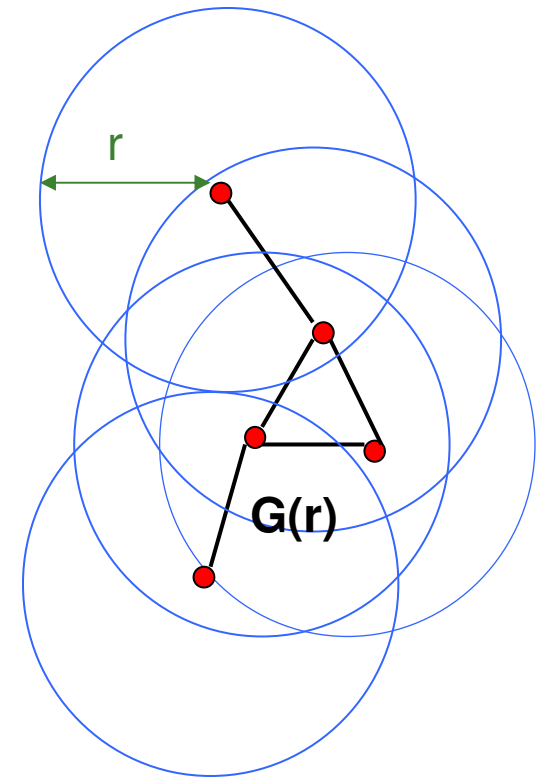
- If successful then

- **transmission rate** = W



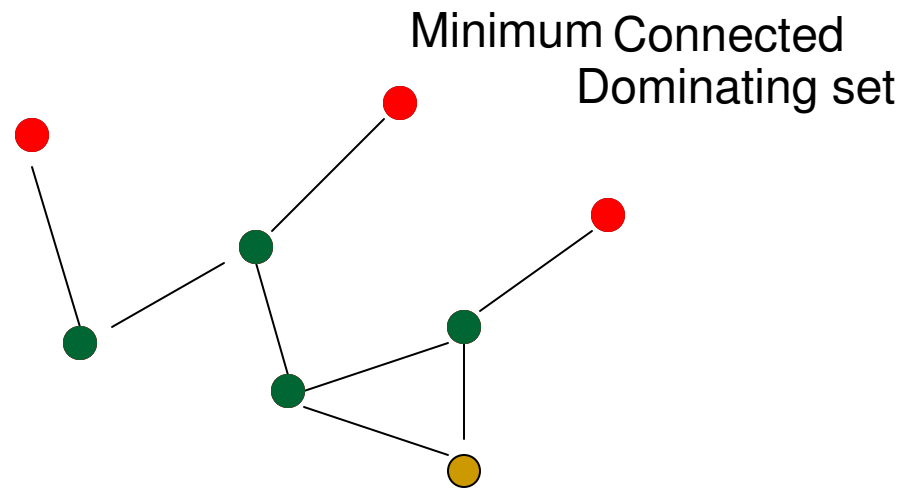
Network Model

- **Geometric graph** model of network
 - **Vertices** of $G(r)$: nodes of network
 - **Edges** of $G(r)$: joins nodes closer than “ r ”
- **Assumption:**
 - **Network is connected**
 - Equivalently: **$G(R)$ is connected**
 - Recall: R = transmission range



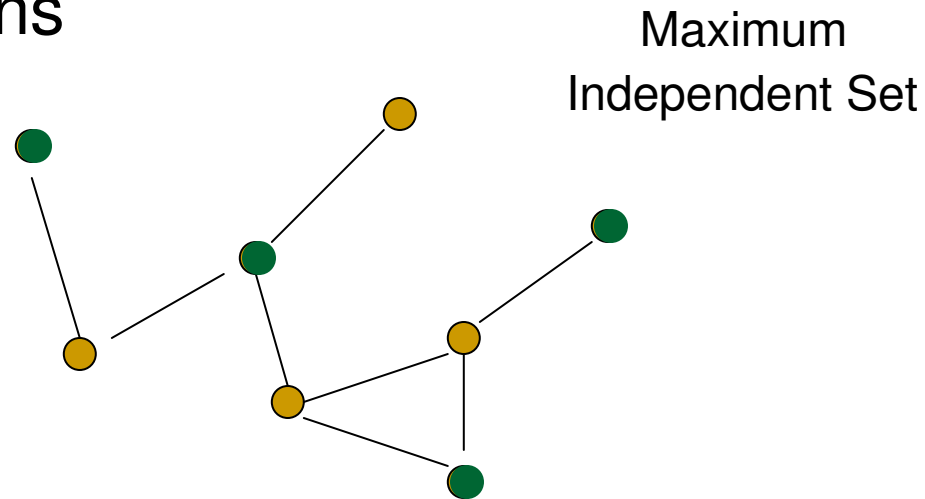
Graph Theory Concepts

- **Dominating Set (DS):**
 - Definition: every network node adjacent to a node of DS.
 - **Connected Dominating Set (CDS):**
 - Set of forwarding nodes for a successful broadcast
 - **Minimum Connected Dominating Set (MCDS):**
 - Optimal number of broadcast transmissions



Graph Theory Concepts

- **Independent Set (IS):**
 - Definition: no two nodes of IS are adjacent.
 - **Maximum Independent Set (MIS):**
 - Maximum number of successful simultaneous txmissions



- **Notation:** MIS(r), MCDS(r) correspond to graph G(r)

2. Broadcast Capacity

Broadcast Capacity

■ Given:

- B : Set of nodes originating the broadcast
- g_k : Fraction of broadcast packets originated by k^{th} node in B
 - $\sum g_k = 1$
- Results will be independent of B and g_k

■ Broadcast capacity:

- maximum rate of generation of broadcast packets such that
- all nodes receive the packets successfully
- ...in a given time

$$\lambda = \{ \max a \mid \lambda_k = g_k \cdot a \text{ is achievable} \}$$

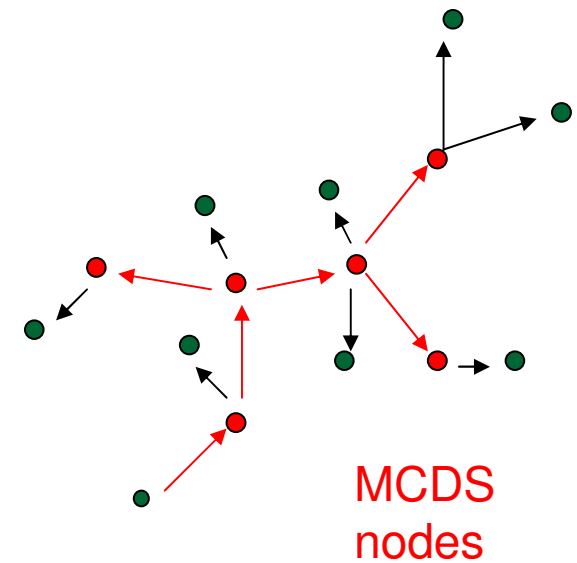
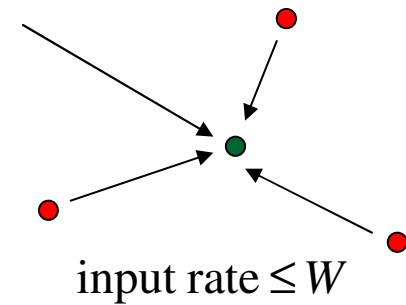
General Bounds on Capacity

■ Thm(1): $\lambda \leq W$

- Transm. Rate W is hard upper bound
- Proof: Each node has to receive

■ Thm(2): $\lambda \geq \frac{W}{\#MCDS(R)+1}$

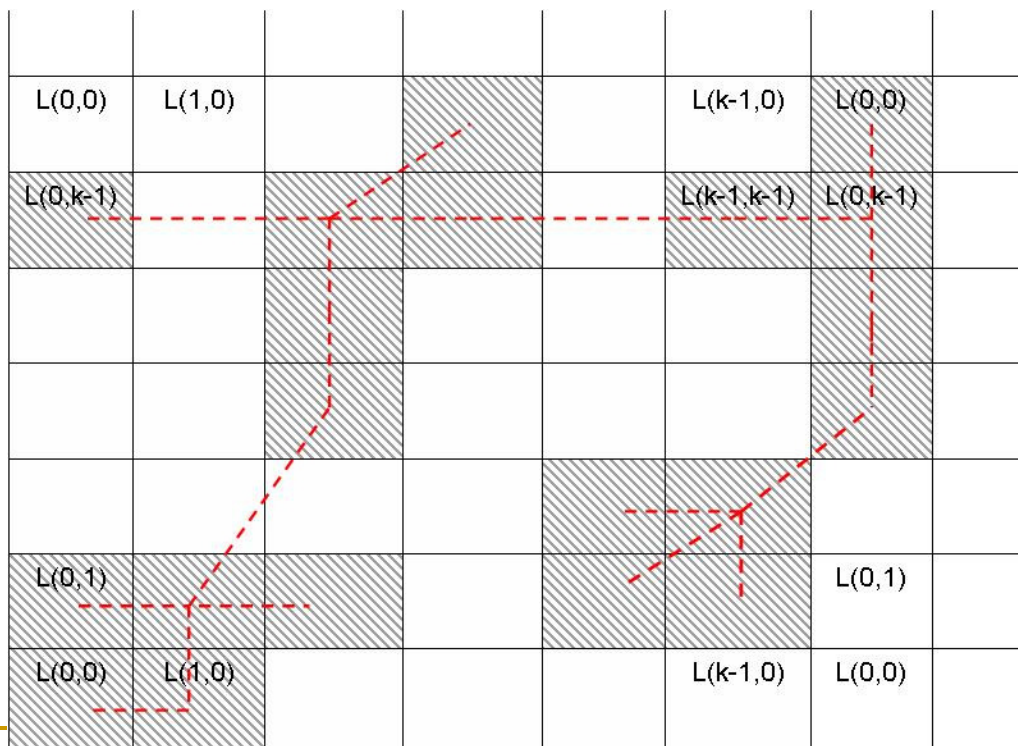
- **Effective** for **small networks** irrespective of interference
- Proof: One transmission at a time over **MCDS nodes**



Improved bounds using Simultaneous Txmissions

■ Thm(3): $\lambda \geq \frac{W}{21 \left[1 + \sqrt{2}(2 + \Delta) \right]^2}$

- In large networks **multiple simultaneous transmissions** give larger throughput



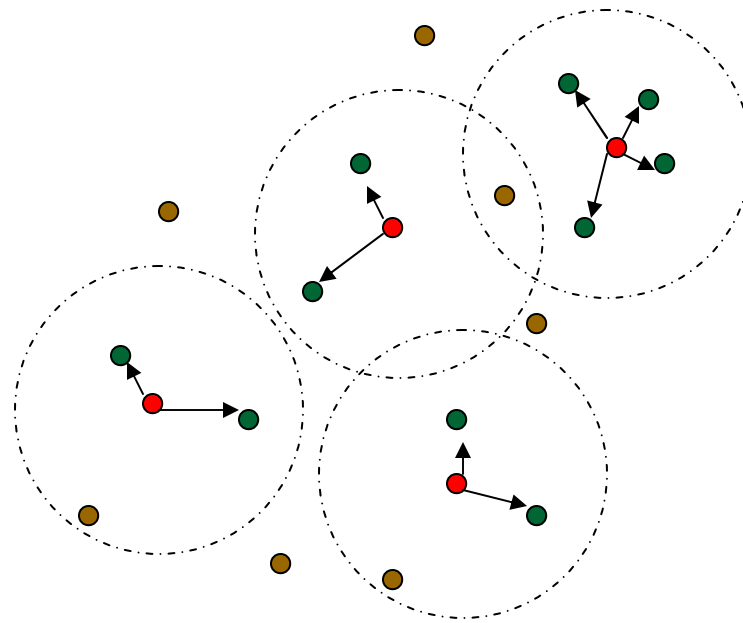
Improved bounds using Simultaneous Txmissions

■ Thm(4): $\lambda \leq W \cdot \frac{\#MIS(\Delta R)}{\#MCDS(R)}$

- Interference limits capacity

Max. simultaneous txmissions

Min. txmissions required



Interference and Capacity

- Thm(6): $c_1 \frac{W}{K(R)} \leq \lambda \leq c_2 \frac{W}{K(R)}$

- Specifies how interference affects capacity
- $K(R)$ is function of topology and interference parameter (Δ)
 - $K(R)$ maximum number of nodes of **MIS(R)** inside interference range of a node

- Thm(8): large homogeneous network

$$c_1 \frac{W}{\max(1, \Delta^d)} \leq \lambda \leq c_2 \frac{W}{\max(1, \Delta^d)}$$

- d = dimension of space

Lessons

Broadcast capacity

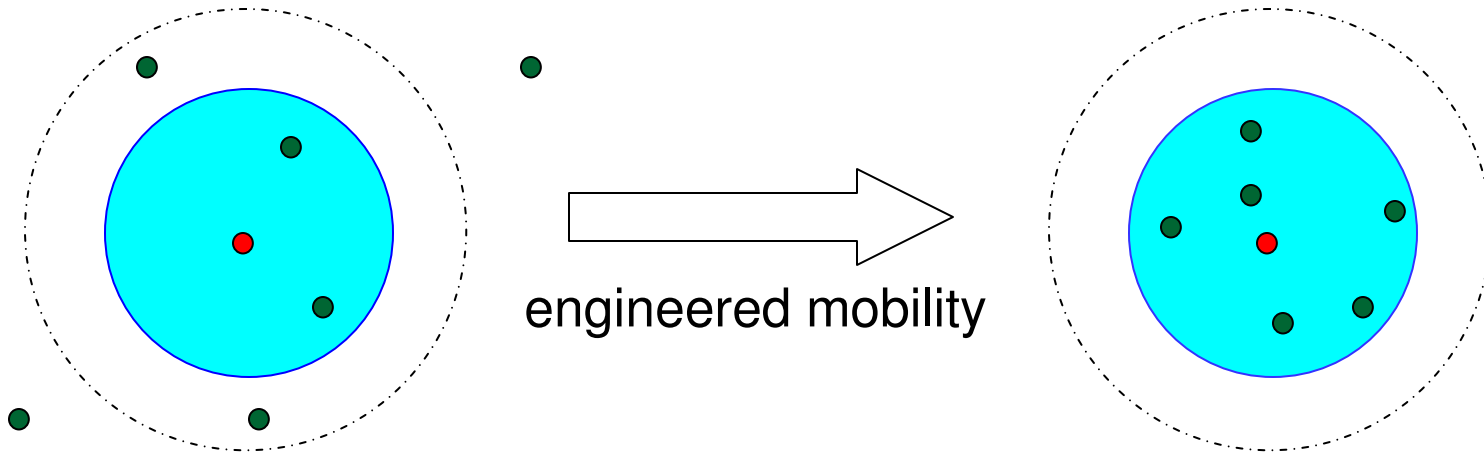
- constant factor of channel capacity
 - depends on **topology** and **interference** parameter.
 - does not change more than a constant when varying
 - **transmission range,**
 - **size of the network,**
 - **number of nodes,** or
 - **originating nodes of broadcast**
-

Does Mobility Increase the Capacity?

- **Mobile network** in d-dimensional space

$$c_1 \frac{W}{\max(1, \Delta^d)} \leq \lambda \leq W$$

- Broadcast capacity increases by at most $\max(1, \Delta^d)$
- Achieve capacity W by **engineering mobility**:



3. Maximum Throughput of Broadcast Schemes

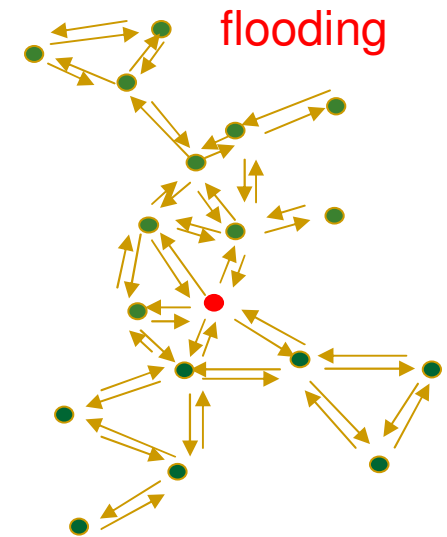
Broadcast Schemes

- **Flooding:**

- All nodes transmit the broadcast packets
- Wasteful in terms of power and capacity

- **Broadcast schemes:**

- **Backbone:** subset of nodes that transmit packets
- Numerous schemes proposed for Ad hoc networks.
- **Efficient schemes:**
 - Keeps number of backbone nodes per unit area bounded.
 - Backbone size within constant factor of MCDS.
- **Inefficient schemes:**
 - All others



Maximum Throughput of Scheme

■ Given:

- B : Set of nodes originating the broadcast
- g_k : Fraction of broadcast packets originated by k^{th} node in B
- Results will be independent of B and g_k

■ Maximum Throughput:

- maximum rate of generation of broadcast packets for a given scheme (S) such that
 - all nodes receive the packets successfully
 - ...in a given time

$$\lambda_S = \{ \max a \mid \lambda_k = g_k \cdot a \text{ is achievable by scheme } S \}$$

Simple Bounds on Maximum Throughput

■ Thm(9): $\lambda_s \leq \lambda \leq W$

■ Thm(10): $\lambda_s \geq \frac{W}{\# \text{Backbone}(R) + 1}$


■ Thm(11): $\lambda_s \leq W \cdot \frac{\# \text{MIS}(\Delta R)}{\# \text{Backbone}(R)}$

Efficient and Inefficient Schemes

- Efficient schemes:
 - Backbone size within constant factor of MCDS.
- Thm(11): For efficient scheme

$$c\lambda \leq \lambda_s \leq \lambda$$

- Thm(12): For inefficient scheme, if $\frac{\#MCDS(R)}{\#Backbone(R)} \rightarrow 0$
then $\lambda_s \rightarrow 0$

Efficiency of the backbone 
maximum throughput within constant factor of broadcast capacity

4. Unicast Capacity vs. Broadcast Capacity

Unicast Capacity

- **Unicast capacity:** Maximum aggregate rate of unicast connections (Gupta & Kumar)

$$\lambda_{\text{unicast}} \bar{L} \leq c \cdot \frac{WA}{\Delta^2 R}$$

- Homogeneous networks:

$$R_{\min} = \sqrt{\frac{A \log(n)}{n}} \quad \Rightarrow \quad \lambda_{\text{unicast}} \leq c \cdot \frac{W}{\Delta^2} \sqrt{\frac{n}{\log(n)}}$$

- Capacity per node:

$$\frac{\lambda_{\text{unicast}}}{n} = c \frac{W}{\sqrt{n \log(n)}} \quad \succ \quad \frac{\lambda_{\text{Broadcast}}}{n} = c \frac{W}{n}$$

Broadcast vs. Unicast

- Unicast capacity strongly depends on network parameters, but **broadcast** capacity **does not**.

	Broadcast	Unicast
Number of nodes (n)	-	$\uparrow \sqrt{n}$
Area (A)	-	$\uparrow \sqrt{A}$
Radio range (R)	-	$\downarrow 1/R$
Mobility	-	$\uparrow n$
Small interference (Δ)	-	$\uparrow 1/\Delta^2$
Large interference (Δ)	$\downarrow 1/\Delta^2$	$\downarrow 1/\Delta^2$
Location & rate of connections	-	\downarrow

Conclusion

Broadcast Capacity

- Up to a **constant factor** the following hold:
 - Broadcast Capacity is equal to **channel capacity**.
 - **Does not change** when varying radio range, number of nodes and the area.
 - **Mobility** does not increase the broadcast capacity
 - **Backbone** must be **efficient** to achieve broadcast capacity
 - **Unlike broadcast capacity**, **unicast capacity** depends on radio range, number of nodes and the area, locations of sources and destinations of flows
-

Question



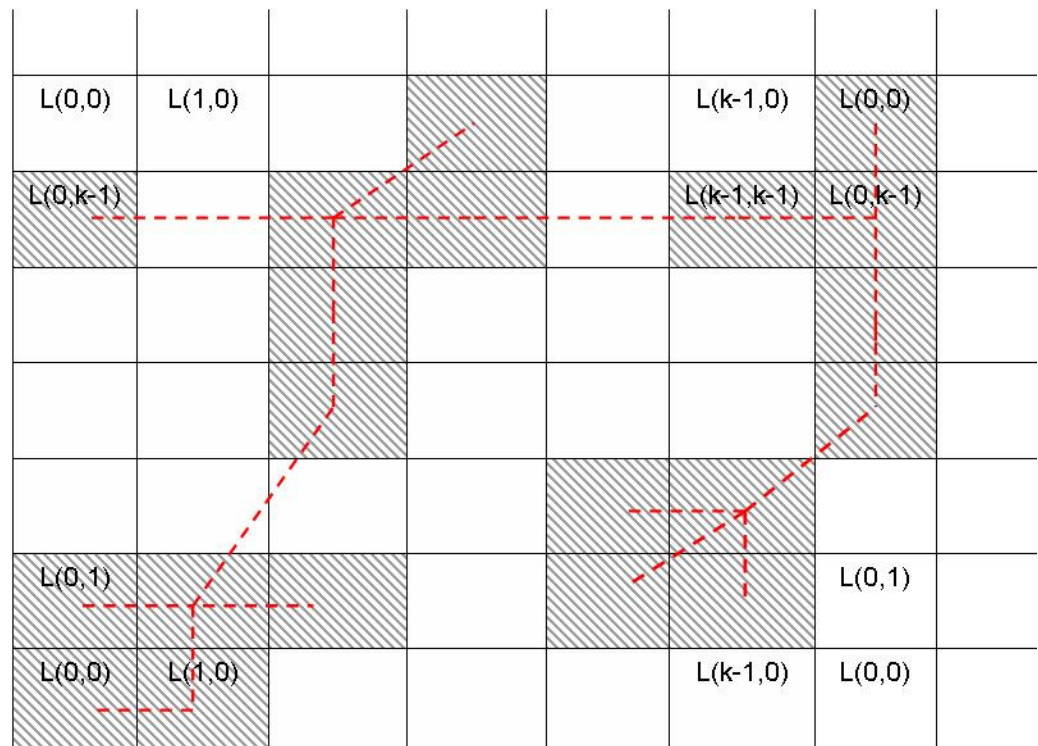
<http://safari.rice.edu>

Future Work

- Study the broadcast capacity of mobile network under different mobility models.
 - Study the broadcast capacity under more accurate channel models.
 - Study the maximum throughput of broadcast schemes under non-idealistic MAC.
-

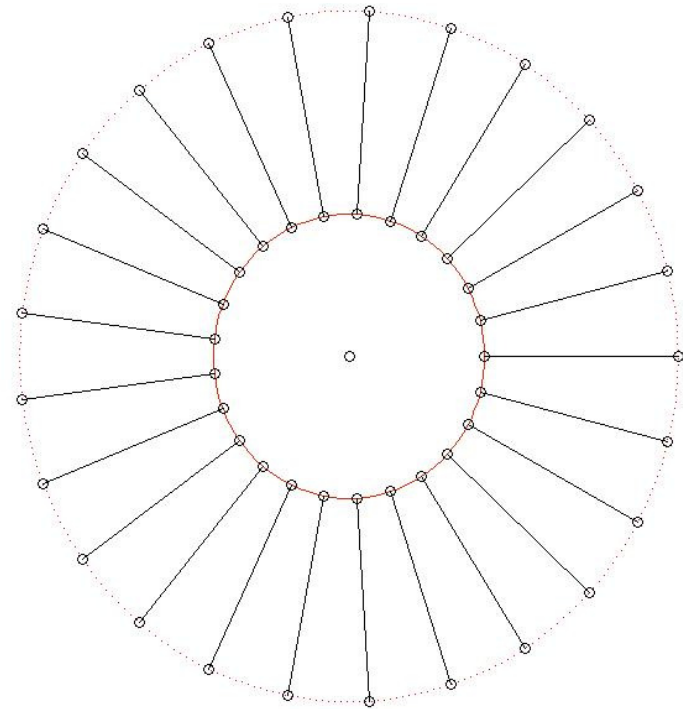
Backup: Broadcasting

- Broadcast scheme that achieves a constant factor of broadcast capacity



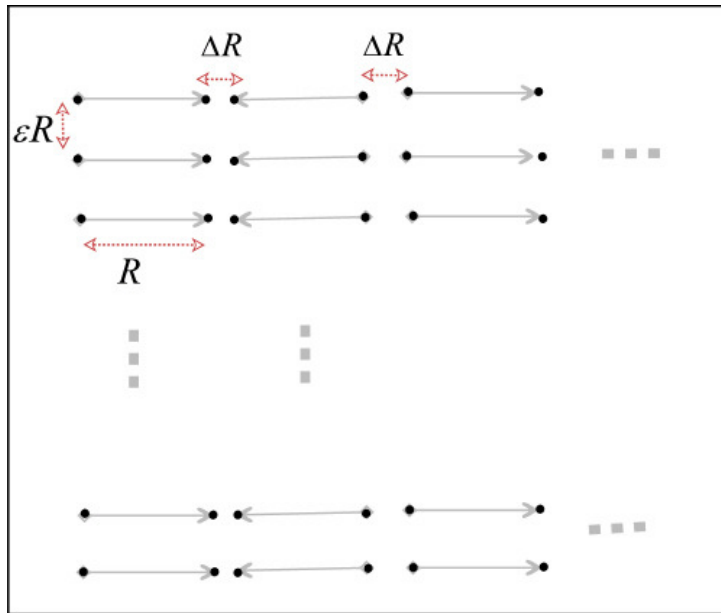
Backup: $\Delta=0$

- High throughput with inefficient backbone under $\Delta=0$ assumption

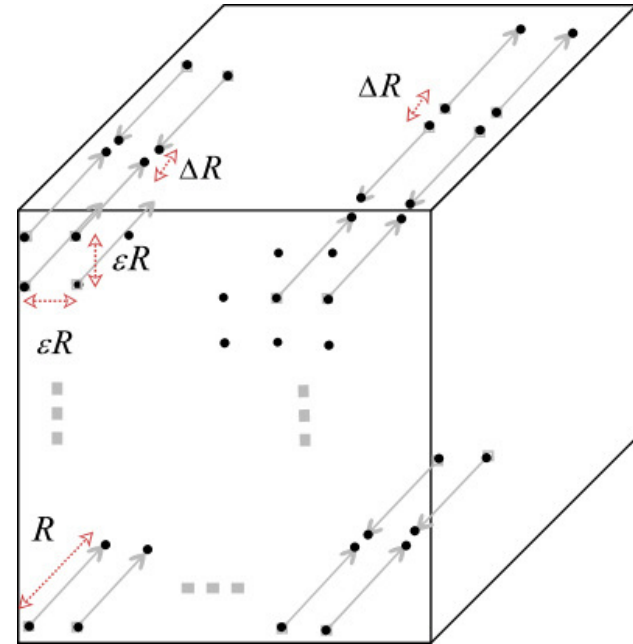


Backup: Unicast Capacity

- For a **small** Δ the unicast would be increased to $O(n)$



$$\varepsilon = \sqrt{3\Delta}$$



Backup: Interference

- Cor(5): If $\Delta > 2$ and $MIS(\Delta R) > 1$
then $\lambda \leq \frac{W}{\lceil \Delta - 2 \rceil}$

Backup: Unicast Capacity of Homogeneous Networks

- **Unicast capacity:** maximum aggregate rate of sending data between random source and destination nodes in the network
 - Upper bound computed as:

$$C_{\text{unicast}} \leq \frac{\text{\# simultaneous transmissions} \cdot W}{\text{\# hops for each packet}}$$
$$= c \cdot \frac{V}{(\Delta R)^d} \cdot W / \frac{\sqrt[d]{V}}{R} = c \cdot \frac{W \sqrt[d]{V^{d-1}}}{\Delta^d R^{d-1}}$$

where V is the volume of the cube contains all nodes.

- If $d > 1$, unicast capacity depends on R , V and Δ
- If $d = 1$, unicast capacity is independent of R , V similar to broadcast capacity.

Backup: Unicast Capacity of Homogeneous Networks

- Minimizing radio range

$$R_{\min} = \sqrt[d]{\frac{V \log(n)}{n}} \quad \longrightarrow \quad C_{\text{unicast}} \leq c \cdot \frac{W}{\Delta^d} \left(\frac{n}{\log(n)} \right)^{\frac{d-1}{d}}$$

where n is number of nodes

- Unicast capacity depends on n
 - A constant factor of C_{unicast} is achievable only for some particular locations of source and destination and flow rate.
-